Coupling Aware Explicit Delay Metric for On-Chip RLC Interconnect for Ramp input

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Abstract— **Recent years have seen significant research in finding closed form expressions for the delay of the RLC interconnect which improves upon the Elmore delay model. However, several of these formulae assume a step excitation. But in practice, the input waveform does have a non zero time of flight. There are few works reported so far which do consider the ramp inputs but lacks in the explicit nature which could work for a wide range of possible input slews. Elmore delay has been widely used as an analytical estimate of interconnect delays in the performance-driven synthesis and layout of VLSI routing topologies. However, for typical RLC interconnections with ramp input, Elmore delay can deviate by up to 100% or more than SPICE computed delay since it is independent of rise time of the input ramp signal. We develop a novel analytical delay model based on the first and second moments of the interconnect transfer function when the input is a ramp signal with finite rise/fall time. Delay estimate using our first moment based analytical model is within 4% of SPICE-computed delay, and model based on first two moments is within 2.3% of SPICE, across a wide range of interconnects parameter values. Evaluation of our analytical model is several orders of magnitude faster than simulation using SPICE. We also discuss the possible extensions of our approach for estimation of source-sink delays for an arbitrary interconnects trees.**

Keywords- Delay Calculation, RLC Interconnect, Moment Matching, Ramp Input.

I. INTRODUCTION

[1]Accurate calculation of propagation delay in VLSI interconnects is critical for the design of high speed systems. Transmission lines effects now play an important role in determining interconnect delay and system performances. Existing techniques are based on either simulation or (closed-form) analytical formulas. Simulations methods such as SPICE give the most accurate insight into arbitrary interconnect structures, but are computationally expensive in total IC design stages. Faster methods based on moment matching techniques are proposed [8-9] but are still too expensive to be used during layout optimization. Thus, Elmore delay [2], a first order approximation of delay under step input, is still the most widely used delay model in

the performance driven synthesis of clock distribution and Steiner global routing topologies. However, Elmore delay cannot be applied to estimate the delay for interconnect lines with ramp input source; this inaccuracy is harmful to current performance-driven routing methods which try to determine optimal interconnect segment lengths and widths (as well as driver sizes). Previous moment-based approaches can compute a response for interconnects under ramp input within a simulation-based methodology [10], but to the best of our knowledge there is no such analytical explicit delay estimation models proposed which is based on the first few moments. Recently, [3] presented lower and upper bounds for the ramp input response; their delay model is the same as the Elmore model for ramp input. Delay estimates for the analytical ramp input model are off by as much as 50% from SPICE-computed delays for 50% threshold voltage [10], and the analytical ramp input model cannot be used to obtain threshold delay for various threshold voltages. Ismail et. al. [4] used Elmore delay as an upper bound on the 50% threshold delay for RLC interconnection lines under arbitrary input waveforms. In this paper, we have proposed a novel and accurate analytical delay estimate for distributed RLC tree interconnects under arbitrary ramp input. To experimentally validate our analysis and delay formula, we model the interconnect lines having various combinations of sources, and load parameters, apply different input rise times, and obtained the delay estimates from SPICE, Elmore delay and the proposed analytical delay model. Over a wide range of test cases, Elmore delay estimates can vary by as much as 100% from SPICE computed delays. As the input rise time increases, Elmore delay deviates even further from SPICE results.

II. BASIC THEORY

A. Central Moments and Transmission Line Response

For a simple input source terminated transmission line we can write the transfer function as,

$$H(s) = \frac{V_O(s)}{V_i(s)} = \frac{1}{sC_L(R_s \cosh(\gamma d) + Z_o \sinh(\gamma d)) + (R_s / Z_o)\sinh(\gamma d) + \cosh(\gamma d)}$$

(1)

Where $\gamma = \sqrt{(R + sL)(sC)}$ is the propagation constant and $Z_o = \sqrt{(R + sL)/(sC)}$ is the

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characteristic impedance of the line. R, L and C are the per-unit-length resistance, inductance, and capacitance of the transmission line, respectively. d is the length of the line. The series resistance is given by, $R_s = R_{dr} + R_{ter}$ where R_{dr} is the driver resistance and R_{ter} the termination resistance. We assume that at the given frequency of interest, the dielectric loss and conductance values are negligibly small. The driver resistance is assumed to be linear. Now the RLC interconnect can be considered as either lossless or lossy.

B. Lossless Interconnect

For an unloaded lossless transmission line driven by a step input, it is well known that the optimal termination resistance is $R_s = Z_0$. With this termination, the ideal signal is the input step delayed by the time-of-flight along the line, and is given by, $T_f = \sqrt{LC}\, d$. The following discussion shows that this ideal response is indeed obtained when the central moments of the impulse response are minimized. For the lossless line in Fig. 1, the transfer function is given by [5-6],

$$H(s) = \frac{1}{(R_s/Z_o)\sinh(\gamma d) + \cosh(\gamma d)} \qquad (2)$$

Where γ and Z_0 are the propagation constant and the characteristic impedance, respectively and are defined as, $\gamma = s\sqrt{LC}$ and $Z_o = \sqrt{L/C}$. For this transfer function, the second and third central moments of the impulse response are symbolically given as:

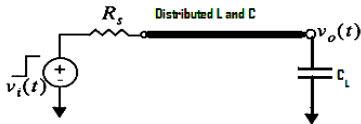$$\mu_2 = -CLd^2 + R_s C^2 d^2 \; and \; \mu_3 = -2R_s C^2 Ld^3 + 2R_s^3 C^3 d^3 \qquad (3)$$

Figure 1.   Lossless Transmission Line

Solving for $\mu_2 = 0$ from (3) yields $\sqrt{L/C}$ and $-\sqrt{L/C}$ as roots. Again solving $\mu_3 = 0$ from (3) yields 0, $\sqrt{L/C}$ and $-\sqrt{L/C}$ as roots. The positive root provides the solution $R_s = Z_0$, Then, the transfer function given as,

$$H(s) = \frac{1}{\sinh(\gamma d) + \cosh(\gamma d)} = e^{-sT_f} \qquad (4)$$

Where $T_f = \sqrt{LC}\, d$ is the time of flight. Then it is easy to show that this transfer function provides the desired ideal waveform at the output of the transmission line

$$v_o(t) = v_i(t - T_f) \qquad (5)$$

From (5), it can be inferred that the ideal impulse response for a lossless transmission line is symmetric

and localized (zero dispersion) about its mean, $\mu = \sqrt{LC}\, d$. Conversely, forcing the impulse response to be symmetric and localized about the mean ensures critical damping.

So from (5) we can write the following equation:

$$V_o(s) = V_i(s) e^{-sT_f} \qquad (6)$$

In case of ramp input,

$$V_i(s) = \frac{V_{DD}}{s^2} \qquad (7)$$

Substituting (7) in (6) we get,

$$V_o(s) = \frac{V_{DD}}{s^2} e^{-sT_f} \qquad (8)$$

Taking inverse Laplace transform of (8),

$$V_o(t) = V_{DD}(t - T_f) u(t - T_f) \qquad (9)$$

For calculation of the time delay we take $V_0(s) = 0.5 V_{DD}$ at time $t = T_D$ and hence substituting in (9), we have,

$$0.5 V_{DD} = V_{DD}(T_D - T_f) u(t - T_f) \qquad (10)$$

So for $t \geq T_f$ the T_D is given as,

$$T_D = T_f + 0.5 \qquad (11)$$

The above equation (11) is our proposed closed form expression for delay for lossless transmission line RLC interconnects system.

C. Lossless Interconnect Considering Mutual Inductance

In order to calculate the exact time delay in two parallel RLC line, we consider the mutual inductance between two inductors as M. Fig. 2 shows two highly coupled transmission line system.
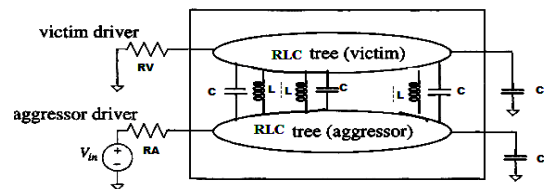

Figure 2.   Highly Coupled Transmission Line

1) Mutual Inductance

The mutual inductance M of two coupled inductances L_1 and L_2 is equal to the mutually induced voltage in one inductance divided by the rate of change of current in the other inductance

$$M = E_{2m} / \left| \frac{di_1}{dt} \right| \qquad (12)$$

$$M = E_{1m} / \left| \frac{di_2}{dt} \right| \qquad (13)$$

If the self induced voltages of the inductances L_1 and L_2 are E_{1s} and E_{2s}, respectively, for

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the same rates of change of the current that produced the mutually induced voltages E_{1m} and E_{2m}, then:

$$M = \left(\frac{E_{2m}}{E_{1s}} \right) * L_1 \qquad (14)$$

$$M = \left(\frac{E_{1m}}{E_{2s}} \right) * L_2 \qquad (15)$$

Combining equations (14) and (15) we get,

$$M = \sqrt{\frac{E_{1m} E_{2m}}{E_{1s} E_{2s}}} * \sqrt{L_1 L_2} = k_M (L_1 L_2)^{1/2} \qquad (16)$$

Where k_M is the mutual coupling coefficient of the two inductances L_1 and L_2. If the coupling between the two inductances L_1 and L_2 is perfect, then the mutual inductance M is:
M = $(L_1 L_2)^{1/2}$

2) Odd Mode

When two coupled transmission lines are driven with voltages of equal magnitude and 180^0 out of phase with each other, odd mode propagation occurs. The effective capacitance of the transmission line will increase by twice the mutual capacitance, and the equivalent inductance will decrease by the mutual inductance [7]. In Fig. 3, a typical transmission line model is considered where the mutual inductance between aggressor and victim connector is represented as M_{12}. L_1 and L_2 represent the self inductances of aggressor and victim nodes, respectively, while C_c, C denotes the coupling capacitance between aggressor and victim and self capacitance, respectively.
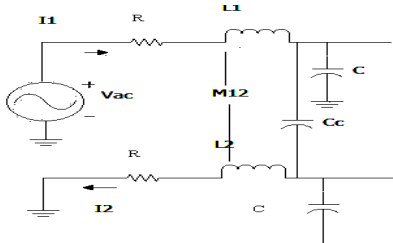

Figure 3.   An Example for Two Parallel Transmission Line Model

Assuming that $L_1 = L_2 = L_0$, the currents will be of equal magnitude but flow in opposite direction [7]. Thus, the effective inductance due to odd mode of propagation is given by,

$$L_{odd} = L_1 - L_2 \qquad (17)$$

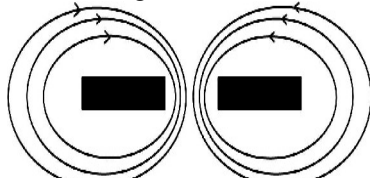The magnetic field pattern of the two conductors in odd-mode is shown in Fig. 4.

Figure 4.   Magnetic Field in Odd Mode

3) Even Mode

When two coupled transmission lines are driven with voltages of equal magnitude and in phase with each other, even mode propagation occurs. In this case, the effective capacitance of the transmission line will be decreased by the mutual capacitance and the equivalent inductance will increase by the mutual inductance [7]. Thus, in even-mode propagation, the currents will be of equal magnitude and flow in the same direction. The magnetic field pattern of the two conductors in even-mode is shown in Fig. 5. The effective inductance due to even mode of propagation is then given by,
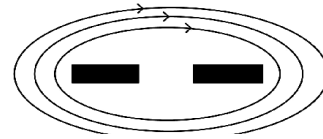
$$L_{even} = L_1 + L_2 \qquad (18)$$

Figure 5.   Magnetic Field in Even Mode

III.   PROPOSED DELAY MODEL

A.   Calculation of Delay for Even Mode

From (1) and for a simple input source terminated transmission line, we can write the transfer function as,

$$H(s) = \frac{V_O(s)}{V_i(s)}$$

$$= \frac{1}{sC_L(R_s \cosh(\gamma_e d) + Z_{oe} \sinh(\gamma_e d)) + (R_s/Z_{oe}) \sinh(\gamma_e d) + \cosh(\gamma_e d)}$$

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$$\qquad (19)$$

Where is the $\gamma_e = \sqrt{(R + s(L+M))(sC)}$ is the propagation constant for even mode and $Z_{oe} = \sqrt{(R + s(L+M))/(sC)}$ is the characteristic impedance for even mode of the line. R, L, M and C are the per-unit-length resistance, inductance, mutual inductance and capacitance parameters of the transmission line, respectively, d is the length of the line, and the series resistance is given by $R_s = R_{dr} + R_{ter}$ where R_{dr} is the driver resistance and R_{ter} the termination resistance. We assume that the dielectric loss and hence the conductance, G to be negligibly small. The driver resistance is assumed to be linear. For an unloaded lossless transmission line driven by a step input, it is well known that the optimal termination resistance is $R_s = Z_{oe}$. With this termination, the ideal signal is the input step delayed by the time-of-flight along the line, is given by,
$$T_{fe} = \sqrt{(L+M)C} \, d$$.

The following discussion shows that this ideal response is indeed obtained when the central moments

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of the impulse response are minimized. For the lossless line in Fig.1, the transfer function is given by,

$$H(s) = \frac{1}{(R_s/Z_{oe})\sinh(\gamma_e d) + \cosh(\gamma_e d)}$$

(20)

Where $\gamma_e = s\sqrt{(L+M)C}$ and $Z_{oe} = \sqrt{(L+M)/C}$. For this transfer function, the second and third central moments of the impulse response are symbolically given as:

$$\mu_2 = -C(L+M)d^2 + R_{s_2}C^2 d^2$$

$$\mu_3 = -2R_s C^2(L+M)d^3 + 2R_{s_3}C^3 d^3$$

$$¿|¿$$

$$¿¿$$

(21)

Solving for $\mu_2 = 0$ from equation (21) yields $\sqrt{(L+M)/C}$ and $-\sqrt{(L+M)/C}$ as roots. Again solving $\mu_3 = 0$ from equation (21) yields 0, $\sqrt{(L+M)/C}$ and $-\sqrt{(L+M)/C}$ as roots. The positive root provides the solution $R_s = Z_{oe}$, Then, the transfer function given as,

$$H(s) = \frac{1}{\sinh(\gamma_e d) + \cosh(\gamma_e d)} = e^{-sT_{f_e}}$$

(22)

Where, $T_{f_e} = \sqrt{(L+M)C}\,d$ is the time of flight. Then it can be easily shown that this transfer function provides the desired ideal waveform at the output of the transmission line is: $v_o(t) = v_i(t - T_{f_e})$

From above, it can be inferred that the ideal impulse response for a lossless transmission line is symmetric and localized (zero dispersion) about its mean, $\mu = \sqrt{(L+M)C}\,d$. Conversely, forcing the impulse response to be symmetric and localized about the mean ensures critical damping.

So from equation (22) we can write the following equation:

$$V_o(s) = V_i(s) e^{-sT_{f_e}}$$

(23)

In case of ramp input,

$$V_i(s) = \frac{V_{DD}}{s^2}$$

(24)

Substituting (24) in (23) we get,

$$V_o(s) = \frac{V_{DD}}{s^2} e^{-sT_{f_e}}$$

(25)

Taking inverse lapalce transform of (25),

$$V_o(t) = V_{DD}(t - T_{fe})u(t - T_{fe})$$

(26)

For the calculation of the time delay we take $V_0(s) = 0.5V_{DD}$ at time t=T_D and hence substituting in (26), we have,

$$0.5 V_{DD} = V_{DD}(T_D - T_{fe})u(t - T_{fe})$$

(27)

So for $t \geq T_{fe}$, T_D is given as,

$$T_D = T_{fe} + 0.5$$

(28)

The above equation (28) is our proposed closed form expression for delay for lossless transmission line RLC tree circuit in even mode and with mutual inductance.

B. Calculation of the Delay in Odd Mode

Again from equation (1) for a simple input source terminated transmission line we can write the transfer function as,

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$¿\,\frac{1}{sC_L(R_s\cosh(\gamma_o d) + Z_{oo}\sinh(\gamma_o d)) + (R_s/Z_{oo})\sinh(\gamma_o d) + \cosh(\gamma_o d)}$$

$$¿|¿$$

$$::$$

(29)

Where, $\gamma_o = \sqrt{(R + s(L-M))(sC)}$ is the propagation constant and $Z_{oo} = \sqrt{(R + s(L-M))/(sC)}$ is the characteristic impedance for odd mode of the line, respectively. R, L, M and C are the per-unit-length resistance, inductance, mutual inductance and capacitance parameters of the transmission line, respectively, d is the length of the line, and the series resistance is given by $R_s = R_{dr} + R_{ter}$. Where R_{dr} is the driver resistance and R_{ter} the termination resistance. We assume the dielectric loss and hence the shunt conductance, *G* to be negligibly small. The driver resistance is assumed to be linear.

For an unloaded lossless transmission line driven by a step input, it is well known that the optimal termination resistance is $R_s = Z_{oo}$. With this termination, the ideal signal is the input step delayed by the time-of flight along the line, is given by, $T_{fo} = \sqrt{(L-M)C}\,d$.

The following discussion shows that this ideal response is indeed obtained when the central moments of the impulse response are minimized. For the lossless line in Fig.1, the transfer function is given by,

$$H(s) = \frac{1}{(R_s/Z_{oo})\sinh(\gamma_o d) + \cosh(\gamma_o d)}$$

(30)

Where $\gamma_o = s\sqrt{(L-M)C}$ is the is the propagation constant and $Z_{oo} = \sqrt{(L-M)/C}$ is the characteristic impedance for this transfer function, the second and third central moments of the impulse response are symbolically given as:

$$\mu_2 = -C(L-M)d^2 + R_{s^2}C^2 d^2$$

$$\mu_3 = -2R_s C^2(L-M)d^3 + 2R_{s^3}C^3 d^3$$

(31)

$$()$$

Solving for $\mu_2 = 0$ from equation (31) yields $\sqrt{(L-M)/C}$ and $-\sqrt{(L-M)/C}$ as roots. Again solving $\mu_3 = 0$ from equation (31) yields 0, $\sqrt{(L-M)/C}$ and $-\sqrt{(L-M)/C}$ as roots. The positive root provides the solution $R_s = Z_{0o}$, Then, the transfer function may be expressed as,

$$H(s) = \frac{1}{\sinh(\gamma_o d) + \cosh(\gamma_o d)} = e^{-sT_{f_o}} \quad (32)$$

Where $T_{f_o} = \sqrt{(L-M)C}\,d$ is the time-of-flight. Then it is easy to show that this transfer function provides the desired ideal waveform at the output of the transmission line is $v_o(t) = v_i(t - T_{f_o})$.

From above, it can be inferred that the ideal impulse response for a lossless transmission line is symmetric and localized (zero dispersion) about its mean, $\mu = \sqrt{(L-M)C}\,d$ conversely, forcing the impulse response to be symmetric and localized about the mean ensures critical damping.

So from (32), we can write the following equation:

$$V_o(s) = V_i(s) e^{-sT_{f_o}} \quad (33)$$

In case of ramp input,

$$V_i(s) = \frac{V_{DD}}{s^2} \quad (34)$$

Substituting (34) in (33) we get,

$$V_o(s) = \frac{V_{DD}}{s^2} e^{-sT_{f_o}} \quad (35)$$

Taking inverse Laplace transform of equation (35)

$$V_o(t) = V_{DD}(t - T_{fo}) u(t - T_{fo}) \quad (36)$$

In order to calculate the time delay we take $V_0(s) = 0.5V_{DD}$ at time $t = T_D$ and hence putting in equation (36), we have,

$$0.5V_{DD} = V_{DD}(T_D - T_{fo}) u(t - T_{fo}) \quad (37)$$

So for $t \geq T_{fo}$ the T_D is given as,

$$T_D = T_{fo} + 0.5 \quad (38)$$

The above equation (38) is our proposed closed form expression for delay for lossless transmission line RLC tree circuit in Odd mode and with mutual inductance.

IV. EXPERIMENTAL RESULTS

In the case of very high frequencies as in GHz scale, inductive effect comes into the important role and it should be included for complete delay analysis. The configuration of circuit for simulation is shown in Figure 6.
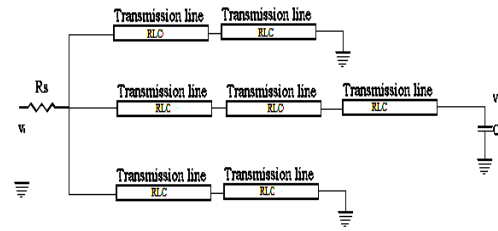

Figure 6. An RLC Tree Example

For each RLCG network source we put a driver, where the driver is a step voltage source followed by a resistor. The results are based on equation (38) for 0.18 μm process. The left end of the first line of Fig. 6 is excited by 1V ramp form voltage with rise/fall times 0.5 ns and a pulse width of 1ns. In table 1, the 50% delay for even mode and the Elmore delay is compared for various values of the driver resistance R_D and the load capacitance C_L when the length of the RLC interconnect is kept constant. In the similar way, in table 2 the 50 % delay for odd mode and the Elmore delay are compared.

TABLE I
EXPERIMENTAL RESULT UNDER RAMP INPUT FOR EVEN MODE

Ex	R_s (Ω)	C_L(fF)	L(μm)	T_{ED} (ps)	Proposed Delay Model (ps)
1	1	10	100	0.1251	0.1342
2	2	50	100	0.1567	0.1576
3	5	750	100	0.4589	0.4765
4	10	1000	100	0.9310	0.9142
5	50	1500	100	0.3920	0.3675
6	100	1500	100	0.5955	0.5762

TABLE II
EXPERIMENTAL RESULT UNDER RAMP INPUT FOR ODD MODE

Ex.	R_s (Ω)	C_L(fF)	L(μm)	T_{ED} (ps)	Proposed Delay Model (ps)
1	1	10	100	0.1065	0.1127
2	2	50	100	0.2597	0.2376
3	5	750	100	0.4943	0.4869
4	10	1000	100	0.9876	0.9792
5	50	1500	100	0.3724	0.3684
6	100	1500	100	0.5732	0.5989

In table 3 and table 4, comparative result of our proposed model delay with the SPICE delay are given in the similar way as we did for the comparison of our proposed model and Elmore delay model discussed above as in table 1 and table2.

V. CONCLUSIONS

In this paper we have proposed an accurate delay analysis approach for distributed RLC interconnect line under ramp input. The use of transmission line model in our study gives a very accurate estimate of the actual delay. We derived the transient response in time domain function of ramp input. We can see that when inductance is taken into consideration, the Elmore approach could result an error of average 10% compared to the actual 50% delay calculated using our approach.

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TABLE III

COMPARATIVE RESULT WITH SPICE UNDER RAMP INPUT FOR EVEN MODE

R_s (Ω)	C_L(fF)	L(μm)	SPICE (PS)	Proposed Delay Model (ps)	% Error
1	10	100	0.1451	0.1342	7.51
2	50	100	0.1595	0.1576	1.19
5	750	100	0.4789	0.4765	0.51
10	1000	100	0.9317	0.9142	1.87
50	1500	100	0.3928	0.3675	6.44
100	1500	100	0.5997	0.58	3.91

TABLE IV.

COMPARATIVE RESULT WITH SPICE UNDER RAMP INPUT FOR ODD MODE

Rs (Ω)	CL(fF)	L(μm)	SPICE (PS)	Proposed Delay Model (ps)	% Error
1	10	100	0.1185	0.1127	4.89
2	50	100	0.2588	0.2376	8.19
5	750	100	0.4994	0.4869	2.5
10	1000	100	0.9843	0.9792	0.51
50	1500	100	0.3792	0.3684	2.84
100	1500	100	0.5787	0.5989	3.49

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